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# IBN ISḤĀQ AL-TŪNISĪ AND IBN MUʿĀDH AL-JAYYĀNĪ ON THE QIBLA

Julio Samsó and Honorino Mielgo

## 1. *Introduction.*

Abū-l-ʿAbbās Aḥmad ibn ʿAlī ibn Isḥāq al-Tamīmī al-Tūnisī was an Tunisian astronomer who lived in the early 13th c.[1] He compiled an astronomical handbook with tables (*zīj*) a manuscript of which (Hyderabad Andra Pradesh State Library no. 298, copied *ca.* 1400), has been discovered by D.A. King. Prof. King kindly provided a complete set of photographs of this precious source[2]. The manuscript is not foliated, but the tables are numbered; therefore, references in this paper will be given to the appropriate number of the chapter of the canons or to the number of the corresponding table.

[1] H. SUTER, *Die Mathematiker und Astronomen der Araber und ihre Werke* (Leipzig, 1900) pp. 142-143; E.S. KENNEDY and D.A. KING, *Indian Astronomy in Fourteenth Century Fez: the Versified Zīj of al-Qusunṭīnī*. "Journal for the History of Arabic Science" 6 (1982), 6-7 (reprint in D.A. KING, *Islamic Mathematical Astronomy*. Variorum Reprints. London, 1986).

[2] He also corrected the English version of this paper and offered a considerable amount of valuable suggestions for which, as well as for many other things, we feel most grateful.

A superficial examination of the *Zīj* shows that, in its present state, it does not represent the actual work of Ibn Isḥāq but rather a compilation, based on it, done perhaps by one of his disciples. The prologue of the canons ascribes to him the compilation of mean motion tables in longitude and anomaly as well as tables of the planetary equations. To these original tables, others have been added that were necessary (*wa uḍīfat ilā jadāwil Ibn Isḥāq mā tadʿū ilay-hi al-ḍarūra*). On the other hand, it seems that Ibn Isḥāq did not write any canons (*rasā'il*) for his tables (*laysa li-Ibn Isḥāq rasā'il*) and the compiler used materials from other tables such as the canons of the *zījes al-Kawr ʿalā al-dawr* and *al-Muqtabis* by Abū-l-ʿAbbās ibn al-Kammād, considered by the compiler as a follower of Abū Isḥāq Ibrāhīm al-Naqqāš, known as al-Zarqiyā[l][3] (Toledo, 11th c.). Other canons were taken by the compiler from the *zīj* called *al-Kitāb al-kāmil fī-l-taʿālīm* written by the *faqīh* Abū-l-Ḥasan ibn ʿAbd al-Ḥaqq al-Ghāfiqī, known as Ibn al-Hā'im al-Išbīlī[4]. The title of this paper is, therefore, inexact, for the materials we are going to use derive from the canons which, as we have seen, do not belong to Ibn Isḥāq's work.

Another remark should also be made here: both the tables and the canons transmit a great quantity of materials derived from the Andalusian astronomical tradition. As for the canons, something has been said in the previous paragraph. Concerning the tables we will only mention that all the mean motion tables include two epoch positions for the beginning of the *Hijra* calculated for the longitudes of Toledo and Tunis and that the tables concerned with the computation of the solar longitude, longitude of the solar apogee and

[3] On Ibn al-Kammād see J. VERNET, *Un tractat d'obstetrícia astrològica* in "Estudios sobre Historia de la Ciencia Medieval" (Barcelona-Bellaterra, 1979), 273-300, and J. SAMSO, *Las Ciencias de los Antiguos en al-Andalus* (Madrid, 1992), 320-324. No exact dates are known for his life: L. RICHTER-BERNBURG, (*Ṣāʿid, the* Toledan Tables, *and Andalusī Science*. "From Deferent to Equant. A Volume of Studies...in Honor of E.S. Kennedy", New York, 1987, pp. 383 and 396 n. 59) considers him to be one of the collaborators of Ibn al-Zarqālluh who worked with him in Cordova ca. 1088. On Ibn al-Zarqālluh/Zarqālī/Zarqiyāl, see J. VERNET in the "Dictionary of Scientific Biography" 14 (New York, 1976), 592-595, and J. SAMSO, *Ciencias de los Antiguos* pp. 147-152, 166-240.

[4] This *zīj* was dedicated, in 1204-05, to the Almohad Caliph Abū ʿAbd Allāh Muḥammad al-Nāṣir (1199-1213). It is extant in the Bodleian Library (Oxford) II,2 ms. 285 (Marsh 618). See J. SAMSO, *Ciencias de los Antiguos* pp. 324-326.



trepidation of the equinoxes derive clearly from the lost work of Ibn al-Zarqālluh the title of which was either *Fī sanat al-šams* ("On the solar year"), or *al-Risāla al-jāmiʿa fī-l-shams* ("A comprehensive epistle on the Sun")[5], as well as from his book on the motion of the fixed stars, extant in a Hebrew translation[6].

## 2. *Andalusian "qibla" materials.*

Chapter 41 of the canons of Ibn Isḥāq's *zīj* deals with the determination of the azimuth of the *qibla* and it is an excellent example of the preservation of Andalusian material in this book. Part of it has little interest such as the description of a quadrant containing a rudimentary sundial of the type known as *balāṭa*: other descriptions of the same instrument are extant in texts ascribed to Qāsim ibn Muṭarrif al-Qaṭṭān (fl. Cordova in the mid 10th c.), Ibn al-Ṣaffār (d. 1034-35) and Maimonides (d. 1204)[7]. When the quadrant is correctly oriented towards the cardinal points, the mid point between East and South will mark the direction of the *qibla* for Ifrīqiya (Tunis). $Q$ = 45° between South and East is a value well documented both in the Andalusian and in the Maghribī tradition[8]

[5] See J. SAMSO, *Ciencias de los Antiguos* pp. 207-208, and J. SAMSO and E. MILLAS, *Ibn al-Bannā', Ibn Isḥāq and Ibn al-Zarqālluh's Solar Theory* in this volume.

[6] See J.M. MILLAS VALLICROSA, *Estudios sobre Azarquiel*. Madrid-Granada, 1943-50, pp. 239-343.

[7] See D.A. KING, *Three Sundials from Islamic Andalusia*. "Journal for the History of Arabic Science" 2 (1978), 387-388, reprinted in D.A. KING, *Islamic Astronomical Instruments*. Variorum Reprints. London, 1987; J. SAMSO, *Ciencias de los Antiguos* pp. 101-104; J. CASULLERAS, *Descripciones de un cuadrante solar atípico en el Occidente Musulmán*, to be published in "Al-Qanṭara".

[8] See KING, *Three Sundials*, pp. 372 and 374, and *A Fourteenth Century Tunisian Sundial for Regulating the Times of Muslim Prayer*. "Prismata. Festschrift für Willy Hartner" (Wiesbaden, 1977), 187-202, also reprinted in KING, *Islamic Astronomical Instruments*. Variorum Reprints. London, 1987. The value $q$ = 45° is also mentioned by Ibn al-Bannā' al-Marrākušī (1256-1321): see R. PUIG, *Al-Šakkāziyya. Ibn al-Naqqāš al-Zarqālluh. Edición, traducción y estudio*. Barcelona, 1986, p. 53; E. CALVO, *La* Risālat al-Ṣafīḥa al-Muštaraka ʿalā al-Šakkāziyya *de Ibn al-Bannā' de Marrākuš*. "Al-Qanṭara" 10 (1989), 29 and 46-47. The same *qibla* value appears finally in the Alfonsine treatise on the use of the astrolabe: cf. R. MARTI and M. VILADRICH, *En torno a los tratados de uso del astrolabio hasta el*

and it reappears in another passage of the same chapter in which the compiler explains how to orient adequately a standard quadrant when the sun crosses the meridian, putting a gnomon (*ᶜamūd*) in the pole (*quṭb* = centre) of the quadrant. Orientation of an instrument is, again, the topic of another passage dealing with the *ṣafīḥa* (*saphaea*), the instrument invented by al-Zarqālluh. In fact, the passage is an almost literal copy of chapter 52 of al-Zarqālluh's treatise on the *ṣafīḥa šakkāziyya*, and exactly the same contents can be found in chapter 40 of the same author's treatise on the use of the *ṣafīḥa zarqāliyya*[9]. Again, the compiler describes the way to orient an astrolabe held horizontally so that the *qibla* direction can be determined on the outer rim of the back of the instrument and marked on the ground. Here, the value $q$ = 30° East of South is specifically ascribed to Cordova and it is easy to check that our compiler is, again, copying an Andalusian source: in this case the direct source are three chapters of the well known treatise on the use of the astrolabe by Ibn al-Ṣaffār (d. 1034-35)[10].

### 3. *Ibn Muᶜādh on the Indian circle.*

We have altered, in fact, the order in which materials are presented in Ibn Isḥāq's chapter on the *qibla*, for at the beginning of it we read the most interesting part: a provisional edition of this long passage appears, here, as *Appendix 1*. It is the Arabic original of chapter 18 of the canons of the *zīj* of Ibn Muᶜādh al-Jayyānī (d. 1093)[11] which are extant, otherwise, only in a Latin translation by Gerard of Cremona (see *Appendix 2* for a transcription of this

*siglo XIII en al-Andalus, la Marca Hispánica y Castilla*. "Nuevos Estudios sobre Astronomía Española en el siglo de Alfonso X" (Barcelona, 1983), 31-32.

9 R. PUIG, *Al-Šakkāziyya* pp. 75 (Arabic ed.) and 162-163 (Spanish translation); R. PUIG, *Los Tratados de Construcción y Uso de la Azafea de Azarquiel* (Madrid, 1987) p. 73.

10 See the edition of the Arabic text of this treatise by J.M. MILLAS VALLICROSA in "Revista del Instituto Egipcio de Estudios Islámicos en Madrid" 3 (1955), pp. 59-61.

11 On Ibn Muᶜāḏ see Y. DOLD-SAMPLONIUS and H. HERMELINK, *al-Jayyānī* in D.S.B. 7 (New York, 1973), 82-83; J. SAMSO, *Ciencias de los Antiguos*, pp. 137-144, 152-166, 240-244. On the date of his death see the remarks by RICHTER-BERNBURG, *Ṣāᶜid* pp. 381 and 395-96 (n. 48).



chapter). This *zīj* is usually known as the *Tabulae Jahen* for it seems to be, essentially, an adaptation of al-Khwārizmī's *zīj* to the coordinates of Jaén, in southern Spain, and the Latin canons were printed in Nuremberg, 1549, under the title *Saraceni cuiusdam de Eris*[12].

The chapter in question deals first[13] with the determination of the meridian line using the so-called "Indian circle". This simple device (see fig. 1) is well known[14] and it consists in drawing a circle (AB) on the ground and erecting a gnomon (OC) of a suitable size on its centre (O): we will mark on the circle the two points (A and B) in which the shadow of the gnomon crosses the circle before and after noon and the meridian line will be DOF which bisects the angle AOB. Ibn Muᶜādh's technique is a little more elaborated for he describes what he calls a *balāṭa*, a technical term which, in al-Andalus, usually means a horizontal sundial[15] but it is applied, here, to a derivation of the Indian circle (see fig. 2): a *balāṭa* (probably a piece of marble or stone) having a smooth equal surface is placed horizontally on the ground. On it we draw an undetermined number of concentric circles: the diameter of the greatest circle will be twelve times the diameter of the smallest circle of the series. A gnomon of a height about one fifth of the diameter of the greatest circle is erected on the common centre[16]. Then, we observe the

12 See H. HERMELINK, *Tabulae Jahen*. "Archives for the History of the Exact Sciences" 2 (1964), 108-112.

13 In the Latin translation; the text in Ibn Isḥāq's *zīj* explains, first how to calculate the *qibla* and, then, the determination of the meridian line.

14 See, for example, E. WIEDEMANN, *Über den indischen Kreis*. "Mitteilungen zur Geschichte der Medizin und Naturwissenschaften" 2 (1912, 252-255; reprinted in WIEDEMANN, *Gesammelte Schriften zur Arabisch-Islamischen Wissenschaftgeschichte* II (Frankfurt, 1984), 666-669. E.S. KENNEDY, *The Exhaustive Treatise on Shadows* II (Aleppo, 1976) pp. 80-90.

15 See the use of this term by Ibn al-Ṣaffār and Maimonides in D.A. KING, *Three Sundials* pp. 367-368 and 387-389; id. CASULLERAS, *Descripciones de un cuadrante solar*; see also J. CARANDELL, *Risāla fī ᶜilm al-ẓilāl de Muḥammad ibn al-Raqqām al-Andalusī* (Barcelona, 1988), p. 222.

16 If these measurements have any meaning, the radius of the smallest circle should correspond to the length of the meridian shadow in the summer solstice at the latitude for which the instrument has been designed. If we consider, for example, that the radius of the greatest circle is $60^p$, the length of the gnomon will be $12^p$ and the radius of the smallest circle $5^p$: this corresponds

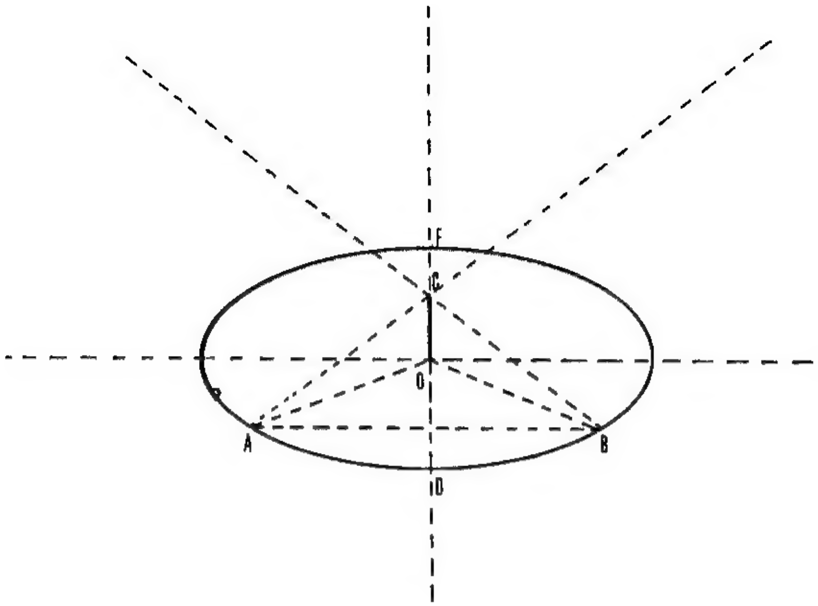


Fig. 1

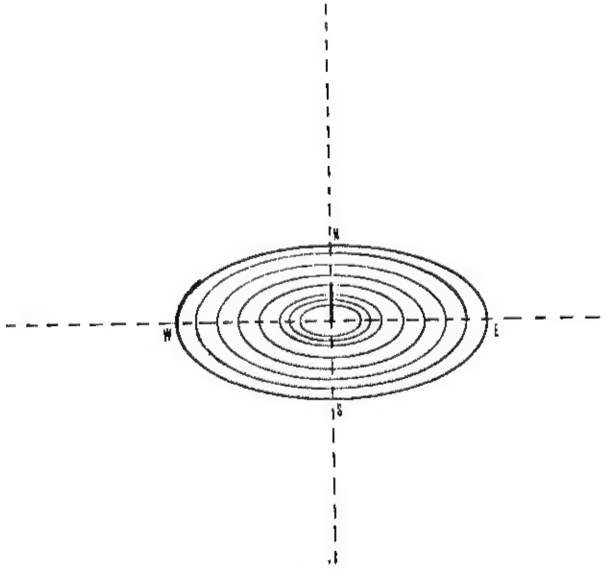


Fig. 2



gnomon's shadow as it crosses each one of the circles. The shadow will intersect the circles twice, before and after noon, except for one — corresponding to the solar maximum altitude at midday— which will be intersected only once. The technique described by Ibn Muᶜādh implies marking the points of intersection of the shadow with each circle, drawing a straight line between the two points corresponding to each circle and determining the midpoint of each one of the segments. If the procedure has been applied correctly all the midpoints, the point corresponding to the minimum shadow and maximum solar altitude for that day (*nuqṭat al-tawaqquf* in Ibn Muᶜādh's terminology) and the common centre of all the circles should lie on a straight line: this will be the meridian.

### 4. *Ibn Muᶜādh and the "method of the zījes".*

#### 4.1 *Introduction.*

The Latin text of the canons of the *Tabulae Jahen* describe the well known "method of the *zījes*" (*al-ṭarīq al-mustaᶜmal fī-l-zījāt*[17]) to determine the azimuth of the *qibla*. The same description appears, ascribed to Ibn Muᶜādh, in the canons of Ibn Isḥaq's *zīj* where it is followed by a worked example for Tunis. This implies that this section was not written by Ibn Muᶜādh, but rather by the compiler of the canons. This is not only the first time the "method of the *zījes*" is documented in the Maghrib but also, to our knowledge, it is the first attestation of an exact method to determine the azimuth of the *qibla* appearing in al-Andalus.

As for the possible sources of Ibn Muᶜādh's treatment of the topic, recent studies have brought attention to the following authors, who lived before the time of the Andalusian astronomer and who dealt with the method of the *zījes*:

to a latitude of about 46° and not to the latitude of Jaén (38° according to Ibn Muᶜād̲ himself in chapter 4 of the Latin canons).

[17] This expression was used by al-Bīrūnī in his *Tahdīd nihāyāt al-amākin li taṣḥīḥ masāfāt al-masākin*. See the critical edition by P. BOULGAKOF in "Revue de l'Institut des Manuscrits Arabes" (Cairo) 8 (1962), 316.

- Ḥabaš al-Ḥāsib (fl. 850)[18].
- Abū-l-Wafā' al-Būzjānī (940-997 or 998) in his *Al-Majisṭī* and in a *zīj* of his, on which some information is known through the 13th c. *al-Zīj al-Šāmil*[19].
- Abū Sahl al-Kūhī (fl. *ca.* 988)[20].
- Al-Bīrūnī (973-1048) at least in three of his works (*Taḥdīd, Maqālīd* and *Qānūn*)[21].
- Kūšyār b. Labbān (*ca.* 971-1029) in his *al-Zīj al-Jāmi*ᶜ[22]
- Ibn Yūnus (d. 1009) in his *al-Zīj al-Ḥākimī*[23].
- Ibn al-Haytham (ca. 965-1039)[24]

18 Ms. Istanbul Yeni Cami 784/2 fol. 150 v- 151 r. The description of the method, without proof, is followed by a worked example in which the *qibla* is calculated for Samarra. Cf. M.T. DEBARNOT, *The* Zīj *of Ḥabash al-Ḥāsib: A Survey of MS Istanbul Yeni Cami 784/2.* "From Deferent to Equant" pp. 35-69 (see especially p. 49); see also M.T. DEBARNOT, *Al-Bīrūnī: Kitāb Maqālīd* ᶜ*ilm al-hay'a. La Trigonométrie Sphérique chez les Arabes de l'Est à la fin du* Xᵉ *siècle.* Damas, 1985, pp. 50, 102 and 254.

19 Cf. J.L. BERGGREN, *On al-Biruni's "Method of the Zijes" for the Qibla.* "Proceedings of the 16th International Congress of the History of Science" C-D (Bucharest, 1981), 237-245 (see p. 241); BERGGREN, *The Origins of al-Bīrūnī's "Method of the* Zījes" *in the Theory of Sundials.* "Centaurus" 28 (1985), 1-16 (especially pp. 5-7); DEBARNOT, *Maqālīd* pp. 102-103; E.S. KENNEDY, *Applied Mathematics in the Tenth Century: Abū'l-Wafā' Calculates the Distance Baghdad-Mecca.* "Historia Mathematica" 11 (1984), 193-206.

20 BERGGREN, *On al-Biruni's "Method of the Zijes"* pp. 237-239; *The Origins* pp. 2-4.

21 AL-BĪRŪNĪ, *Kitāb Taḥdīd nihāyāt al-amākin li-taṣḥīḥ masāfāt al-masākin.* Ed. P. BOULGAKOF in "Revue de l'Institut des Manuscrits Arabes" (Cairo) 8 (1962), 206-209 and 284-286; E.S. KENNEDY, *A Commentary upon Bīrūnī's Kitāb Taḥdīd al-Amākin* (Beirut, 1973), pp. 128-130 and 211-214; DEBARNOT, *Maqālīd* 252-257; AL-BĪRŪNĪ, *Kitāb al-Qānūn al-Mas*ᶜ*ūdī.* Hyderabad, 1954-56, II, pp. 522-525; D.A. KING, *Ḳibla.* "Encyclopédie de l'Islam" V (Leiden-Paris, 1986), 88-89.

22 BERGGREN, *The Origins* pp.7-8.

23 D.A. KING, *The Astronomical Works of Ibn Yūnus.* Ph.D. Dissertation presented at Yale University in 1972 (chapter 28); BERGGREN, *The Origins* pp. 8-10 and 13.

24 D. A. KING, *The Earliest Islamic Mathematical Methods and Tables for Finding the Direction of Mecca.* "Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften" 3 (1986), 82-149 (see p. 116). King also studies (pp. 112-115) an early Abbasid method for finding the *qibla* which is obviously related to the "method of the *zījes*". We do not take it into consideration here because it could not be considered as Ibn Muᶜāḏ's source.



Out of the works of these seven authors which are the possible sources of our Ibn Muᶜādh, only four texts have been available to us: the *zīj* of Ḥabaš and the three aforementioned books of al-Bīrūnī. We know the rest through indirect references. Our conclusions will be, therefore, provisional until more materials are published on this topic. We would like, however, to remember here that Ibn Muᶜādh wrote a book on spherical trigonometry, the *Kitāb majhūlāt qisī al-kura*[25] that bears witness to his knowledge of the new trigonometry developed in the Muslim East in the second half of the 10th c. and beginning of the 11th[26]. From this point of view he is clearly an exception within the Andalusian 11th century, for the kind of authors who contributed most to this renewal of trigonometry (al-Bīrūnī is a good example) do not seem to have been known in al-Andalus[27]. Ibn Muᶜādh's "exceptional" knowledge of these authors should be remembered here, for many of them also appear in the aforementioned list as astronomers interested in the "method of the *zījes*".

## 4.2 *Ibn Mu*ᶜ*ādh's formulation and the worked example.*

We will briefly discuss Ibn Muᶜādh's words together with the worked example following fig. 3 in which we have tried to superimpose the necessary materials to compare, in our commentary, the two proofs of the method as explained by al-Bīrūnī in the *Taḥdīd* (T) and the *Maqālīd* (M), on one side, and in the *Qanūn* (Q) on the other. The letters are those used in *Q* to which others have been added when necessary. In this figure:

TSOAG is the local meridian,
ACLG is the local horizon,
MTH is the meridian of Mecca,
KLEI is the horizon of Mecca,
T is the north pole of the equator,

25 Edited and translated by M.V. VILLUENDAS (Barcelona, 1979).

26 J. SAMSO, *Notas sobre la trigonometría esférica de Ibn Mu*ᶜ*āḏ.* "Awrāq" 3 (1980), 60-68.

27 See RICHTER-BERNBURG, *Ṣā*ᶜ*id*, especially pp. 380-385.

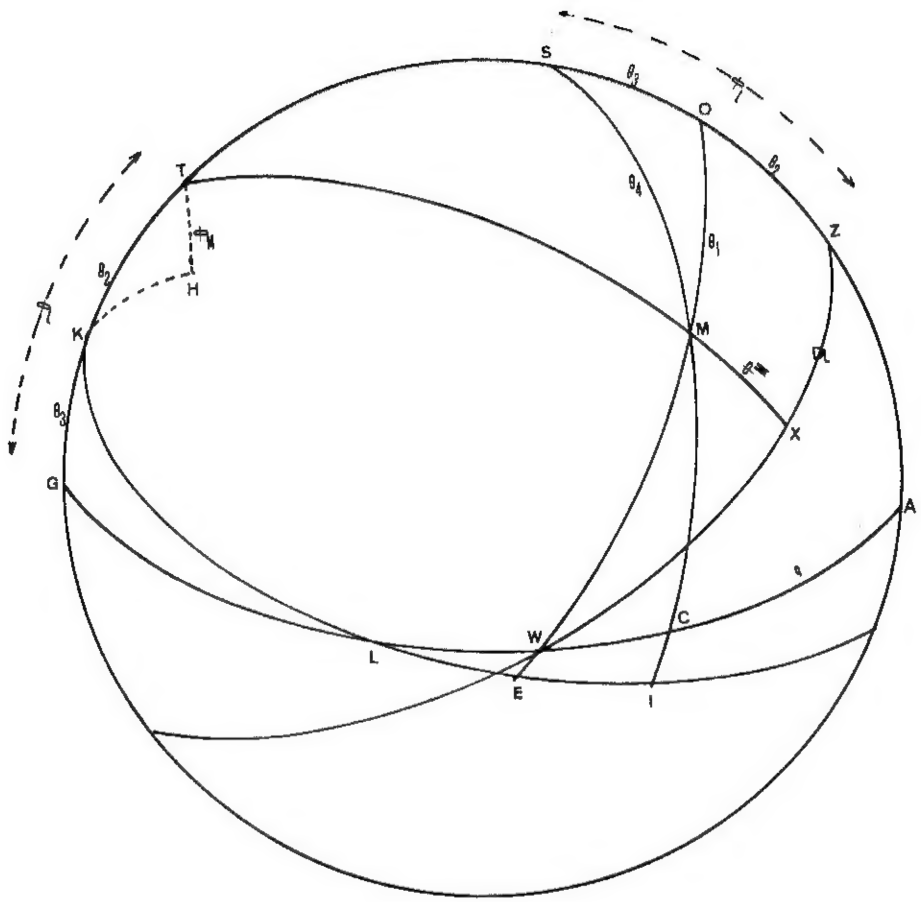


Fig. 3



S is the local zenith,
M is the zenith of Mecca,
SMCI is an arc of a great circle passing through the zenith of our locality and through the zenith of Mecca,
K (intersection of the local meridian with the horizon of Mecca) is the pole of the arc of a great circle EWMO which passes through the zenith of Mecca and through the West point of the local horizon.

In our exposition, we will follow King in calling the four auxiliary arcs $\theta_1$, $\theta_2$, $\theta_3$ and $\theta_4$[28]. We also adopt the usual convention:

$$\mathrm{Sin}\ \theta = 60 \sin \theta$$
$$\mathrm{Cos}\ \theta = 60 \cos \theta$$

Numbers between square brackets [ ] correspond either to differences between the numerical results of the manuscript and our own recomputation or to restorations of the text.

1. Computation of $\theta_1$ (= MO) which Ibn Muᶜādh calls *al-faḍla al-ṭūliyya* or *al-ᶜamūd* (Lat. *superfluitas longitudinis, perpendicularis*):

$$\mathrm{Sin}\ \theta_1 = \mathrm{Cos}\ \varphi_M\ \mathrm{Sin}\ D_l/60$$

for $\varphi_M$ = latitude of Mecca (*Metra* in the Latin translation)
$D_l$ = difference of longitudes between Mecca and our locality.

This results, in *T* and *M*, from the application of the rule of four to right angled triangles TOM and TZX. In *Q*, al-Bīrūnī applies the sine law to the right angled triangle TOM. MO becomes, therefore, known as well as angle OKE = 90° + MO.

In the worked example, the compiler of Ibn Isḥāq's *zīj* establishes that the longitude of Mecca is 77°, a standard value which implies the use of the base meridian of the Fortunate Islands. Ibn Muᶜādh mentions 67° and 67;30° using, in both, the base meridian of

[28] See the edition Hyderabad, 1954, II, 522-525. This passage has been discussed by KING, *Ḳibla* in "E.I." V, 88-89; see also BERGGREN, *The Origins* (cf. especially pp. 13-14).

the Western African Coast. The longitude of Tunis employed in the computation is 41;45°[29]. Therefore:

$D_l$ = 35;15°
Sin $D_l$ = 34;38$^p$

The compiler also states:

Cos $\varphi_M$ = 55;46$^p$[30]

Then:

Sin $\theta_1$ = Cos $\varphi_M$ Sin $D_l$ / 60 = 55;46$^p$ 34;38$^p$ / 60 = 32;11,23$^p$
$\theta_1$ = arcSine 32;11,23 = 32;27°.

2. Computation of $\theta_2$ (OZ in *T* and *M*, TK in *Q*) called by Ibn Muᶜādh *al-buᶜd min muᶜaddil al-nahār* (Lat. *longitudo ab equatore diei*):

Sin $\theta_2$ = 60 Sin $\varphi_M$ / Cos $\theta_1$

This results, in *T* and *M*, either from the application of the sine law to the right angled triangle WMX, or the rule of four to WMX and WOZ. In *Q*, the sine law is applied to the right angled triangle THK in which Sin K = Cos $\theta_1$.

In the worked example:

Cos $\theta_1$ = Cos 32;27° = 50;38$^p$
Sin $\varphi_M$ = [Sin 21;40°] = 22;9$^p$
and Sin $\theta_2$ = 60 Sin $\varphi_M$ / cos $\theta_1$ = 1329 / 50;38$^p$ = 26;15$^p$

[29] A value which appears in al-Khwārizmī and Abū-l-Ḥasan ᶜAlī al-Marrākušī. See E.S. and M.H. KENNEDY, *Geographical Coordinates of Localities from Islamic Sources* (Frankfurt, 1987) p. 363.

[30] We have corrected the 105;46$^p$ of the manuscript. 55;46$^p$ corresponds to a latitude of Mecca of 21;40°, a very common value derived very early which appears in the Latin translation of Ibn Muᶜāḏ's canons. Ibn Muᶜāḏ's Arabic and Latin texts mention also 21;30°, a more accurate value which appears frequently in the Kennedys' colletion, though normally in sources later than our author. Our correction is confirmed later by the text itself which establishes sin $\varphi_M$ = 22;09$^p$ a value that corresponds precisely to $\varphi_M$ = 21;40°.



$\theta_2$ = arcSine 26;15$^p$ = 25;57°.

3. Computation of $\theta_3$ (SO in *T* and *M*, KG in *Q*), *buᶜd al-balad* (Lat. *longitudo regionis*):

$\theta_3$ is the difference between $\theta_2$ and the local latitude ($\varphi_L$). If $\theta_2$ > 90°, then $\theta_3$ = $\theta_2$ + $\varphi_L$.

In our worked example instead of
$\theta_3$ = $\varphi_L$ - $\theta_2$

we have: 90°- $\theta_3$ = (90° - $\varphi_L$) + $\theta_2$ = 53;20° + 25;57° = 79;17°

The compiler is, therefore, using 36;40° for the latitude of Tunis, a value quoted in al-Khwārizmī's *Geography*.

4. Computation of $\theta_4$ (= SM), *masāfa mā bayna baladi-ka wa-Makka* (Lat. *spacium quod est inter regionem tuam et Metram*).

Cos $\theta_4$ = Cos $\theta_3$ Cos $\theta_1$ / 60

This results, in *T* and *M*, from the application of the rule of four to the right angled triangles WMC and WOA. In *Q*, the same rule of four is applied to KSI and KOE:

Sin KS / Sin SI = Sin KO / Sin OE
where: SI = $\theta_4$ + 90°
KS = 90° - $\theta_3$
OE = 90° + $\theta_1$

In the worked example:

Cos $\theta_3$ = Sin 79;17° = 58;57,13$^p$ [48;57,13$^p$ in the manuscript]
Cos $\theta_1$ = 50;38$^p$ (see stage 2 of the computation)

Therefore:

Cos $\theta_4$= Cos $\theta_3$ cos $\theta_1$ / 60= 58;57,13$^p$ 50;38$^p$ / 60= 49;44,50$^p$

a result which is exact if we round $58;57;13^p$ to $58;57^p$.

$\theta_4$ = arcCos $49;44,50^p = 33;56^p$ [-3']

5. Computation of $q$ (= AC), azimuth of the *qibla*:

Sin q = 60 Sin $\theta_1$ / Sin $\theta_4$

In *T* and *M*, this is proved applying the rule of four to right angled triangles SOM and SAC. In *Q* in which al-Bīrūnī uses R = 1, we obtain a different expression:

sin LG = cos $\theta_1$ sin $\theta_3$ / sin $\theta_4$

applying the sine rule to triangle LKG in which

L = $\theta_4$
sin K = cos $\theta_1$

It is easy to show that *LG* = *90°* - *q* if we bear in mind that

AW = CL = GW = 90°.

The last stage in the computation of the worked example is corrupt in the manuscript, but it can be restored without difficulty:

Sin q = 60 Sin $\theta_1$ / Sin $\theta_4$ = [$60^p$] $32;11,23^p$ / [$33;29,37^p$]

in which *Sin* $\theta_1$ = *32;11,23*$^p$ appears, in the text, in stage 1 of the computation, while *Sin* $\theta_4$ = *33;29,37*$^p$ is our own computation of the *Sine* of $\theta_4$ = 33;56°.

According to the manuscript:

Sin q = $73;34^p$

an obvious mistake. The manuscript also states that *q* = *73;34°*. We suggest, therefore:

Sin q = [$60^p$] $32;11,23^p$ / [$33;29,37^p$] = [$57;39,50^p$]



whence q = 73;[57],34°
90° - q = 16;[2],2[6]

instead of the 16;24° of the text.

*5. On the possible sources used by Ibn Muᶜādh.*

It is extremely difficult to establish which of these sources could have been used by Ibn Muᶜādh but a few remarks can be made. We should, first of all, reject al-Bīrūnī's *Qānūn* because, unlike Ibn Muᶜādh, he uses *r=1* and we have seen that the final stage in the computation is different.

Abū-l-Wafā' al-Būzjānī's *Al-Majisṭī* can also be disqualified for it introduces certain improvements in the calculation that are not to be found in Ibn Muᶜādh. We cannot reject the rest of the aforementioned authors. It is not without interest to compare the terminology used by them for the four auxiliary arcs $\theta_1$, $\theta_2$, $\theta_3$ and $\theta_4$:

$\theta_1$ (=MO): *al-ṭūl al-muᶜaddal* (modified longitude) in Ḥabaš, Bīrūnī (*T* and *M*); *taᶜdīl al-ṭūl* (longitude adjustment) in Abū-l-Wafā', al-Kūhī and Kūšyār; *al-buᶜd fī-l-madār* ("distance measured on the parallel/circle") in Bīrūnī (*Q*). Ibn Muᶜādh uses *al-faḍla al-ṭūliyya* ("difference in longitude", Lat. *superfluitas longitudinalis*) and *al-ᶜamūd* ("the perpendicular", Lat. *perpendicularis*). Obviously $\theta_1$ is not $D_I$ but it is the arc MO opposite to angle T (= $D_I$) in triangle MOT. MO is also an arc of a great circle through the zenith of Mecca perpendicular to the local meridian.

$\theta_2$ (=OZ=TK): *ᶜarḍ Makka al-muᶜaddal* ("modified latitude of Mecca) in Ḥabaš; *ᶜarḍ baladi-nā muᶜaddalan bi-ufq dhālika-l-balad* ("latitude of our own locality modifided for the horizon of the other locality) in al-Bīrūnī (*Q*); *al-ᶜarḍ al-muᶜaddal* ("the modified latitude") in al-Bīrūnī (*T* and *M*); *taᶜdīl al-ᶜarḍ* ("latitude adjustment") in Abū-l-Wafā', al-Kūhī and Kūšyār. Ibn Muᶜādh is again independent and uses *al-buᶜd min muᶜaddil al-nahār* ("distance from the equator", Lat. *longitudo ab equatore diei*) which describes the position of point O distant from the equator WXZ by the amount $\theta_2$ (=OZ).

$\theta_3$ (=SO=KG): without any name in Ḥabaš and al-Bīrūnī (*T* and *M*); *ta<sup>c</sup>dīl al-<sup>c</sup>arḍ* ("latitude adjustment") in al-Bīrūnī (*Q*); *al-<sup>c</sup>arḍ al-mu<sup>c</sup>addal* ("modified latitude") in Abū-l-Wafā', al-Kūhī and Kūšyār. Ibn Mu<sup>c</sup>ādh uses *bu<sup>c</sup>d al-balad* ("distance of the locality", Lat. *longitudo regionis*) which, again, describes the position of point O distant from the zenith of our locality (S) by the amount $\theta_3$ (=SO).

$\theta_4$ (=SM): *al-jayb al-awwal* ("the first sine") is used by Ḥabaš to denote Cos $\theta_4$; *al-masāfa bayn al-baladayn* ("distance between the two localities") in al-Bīrūnī (*Q*); *al-masāfa bayn al-balad wa-bayn Makka* ("distance between our locality and Mecca") in al-Bīrūnī (*T*); in *M* al-Bīrūnī shows that he is aware of the fact that MS is the distance between the two localities but does not use such an expression and calls $\theta_4$ *tamām irtifā<sup>c</sup> Makka fī baladi-nā* ("complement of the altitude of Mecca over our locality", SM = 90°-MC). We have no information on the rest of the aforementioned authors. As for Ibn Mu<sup>c</sup>ādh, we have here something meaningful, for he uses, like al-Bīrūnī (*T* and *Q*) *al-masāfa mā bayn baladi-ka wa-Makka* ("distance between your locality and Mecca", Lat. *arcus spacii quod est inter regionem tuam et Metram*). In the *Taḥdīd*, al-Bīrūnī uses the "method of the *zījes*" to establish the distance between two localities and only Abū-l-Wafā' al-Būzjānī appears, in the literature quoted in this paper, to have done a similar thing[31]. As a pure hypothesis, until more information is gathered on the subject, we suggest that Ibn Mu<sup>c</sup>ādh might have known al-Bīrūnī's *Taḥdīd*. The expressions used by Ibn Mu<sup>c</sup>ādh to describe $\theta_2$ and $\theta_3$ agree well with the configuration used by al-Bīrūnī in *T* (and in *M*) to prove the "method of the *zījes*".

Another small detail points in the same direction. As Berggren has remarked, the method requires to establish which is the endpoint (north or south) from which the *qibla* has to be measured and which is the direction (east or west) of the *qibla*. Only two of the aforementioned authors give criteria for this purpose: the *al-Zīj al-Šāmil* (which is essentially based on a lost *zīj* of Abū-l-Wafā') and al-Bīrūnī in the three books we have been using. All of them establish that the azimuth has to be measured from

> the south point of the horizon if $\theta_2 < \varphi_L$
> and from the north point if $\theta_2 > \varphi_L$

something which is fairly obvious in the construction used by al-Bīrūnī in *T* and *M*, but not so obvious in *Q*: if $\theta_2 < \varphi_L$, then the arc of the great circle OMWE will be, like in fig. 3, to the south of the prime vertical SW (not drawn in fig. 3). Otherwise, if $\theta_2 > \varphi_L$, OMWE will be placed towards the north of the prime vertical. Obviously the azimuth will be eastern or western according to the eastern or western situation of Mecca in relation to our locality. There is nothing new in Ibn Mu<sup>c</sup>ādh although, once again, his source can be al-Bīrūnī's *T* or Abū-l-Wafā''s lost *zīj*. We should remark, however, that Ibn Mu<sup>c</sup>ādh introduces a new consideration, which we have been unable to find in any of his predecessors: namely, that the azimuth of the *qibla* will always be northern when the difference of longitudes between Mecca and our locality is greater than 90°. This, like the previous criteria is valid for northern latitudes although it has little practical application, for only localities placed in the very Far East will have longitudes so that $D_1 > 90°$. The two basic cases considered by al-Bīrūnī and Abū-l-Wafā' together with the novelty apparently introduced by Ibn Mu<sup>c</sup>ādh are represented in fig. 4. Here ABCD corresponds to the parallel the distance of which from the equator equals the latitude of Mecca, SWN is the local horizon and Z the local zenith. If the zenith of Mecca lies on arc DC, then the azimuth will be southern and $\theta_2 < \varphi_L$; when it lies on arc CB, the azimuth will be northern and $\theta_2 > \varphi_L$; finally, when it lies on arc AB, the azimuth will also be northern and $D_1 > 90°$.

[31] See KENNEDY, *Applied Mathematics* p. 194.



## 6. *Summary and conclusions.*

We have analysed the chapter on the *qibla* in the canons of the so-called *Zīj* of Ibn Isḥāq and we have established clearly that this chapter is the result of a compilation of various Andalusian sources. One of them (see *Appendix 1* and *2*) is the corresponding chapter of Ibn Mu<sup>c</sup>ādh's *Tabulae Jahen*, the canons of which are extant in

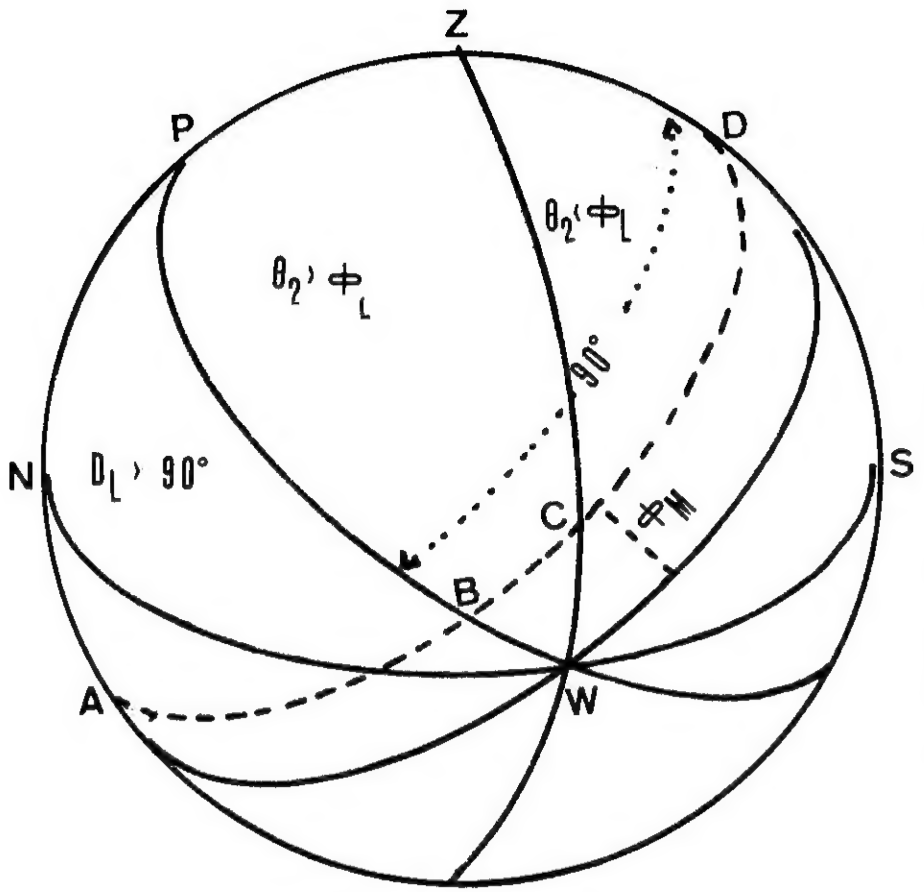


Fig. 4



Gerard of Cremona's Latin translation. This chapter of Ibn Muᶜādh's *zīj* contains a modification of the method of the Indian circle to determine the meridian line, as well as a description of the "method of the *zījes*" for finding the *qibla*. We have observed that it is the first time an exact solution to the problem of the *qibla* is documented in al-Andalus. On the other hand, the terminology used by Ibn Muᶜādh to refer to the four auxiliary arcs used in the procedure appears to be original, which suggests that the Andalusian astronomer did not limit himself to copying the instructions from his source but that he understood correctly the method. This is, perhaps, a reason to reject the *zīj* of Ḥabaš as the source of Ibn Muᶜādh: the terminology used is entirely different and it contains no proof of the method. As for the other sources, we can only hint at the possibility that Ibn Muᶜādh might have been using al-Bīrūnī's *Taḥdīd*, for he is aware of the fact that one of the stages of the computation leads to the distance between two localities measured on an arc of a great circle. Finally, like al-Bīrūnī and Abū-l-Wafā', Ibn Muᶜādh gives the criteria to establish whether the azimuth will be northern or southern, adding a new consideration to those of his predecessors, a feature which, once again, proves that he had a sound understanding of the method. This accords with our assessment of Ibn Muᶜādh as a competent mathematician.

## APPENDIX 1: FROM CHAPTER 41 OF MANUSCRIPT HYDERABAD ANDRA PRADESH STATE LIBRARY 298.

امّاالجهات الأربع والقبلة بمذهب القاضي ابي عبد الله بن معاذ فهو أن تعرف طول مكّة و هو سبع وستّون درجة من المغرب وقد تزيد نصف درجة وعرضها احد وعشرون درجة ونصف وقد تزيد نصف درجة ايضًا واعلم طول بلدك وعرضه ثمّ اضرب جيب تمام [ عرض ] مكّة في جيب ما بين الطولين واقسم المجتمع على ستّين فما خرج قوّسه تكن الفضلة الطولية وهي العمود فاحفظها واجعل جيب تمامها امامًا واضرب جيب عرض مكّة في ستّين واقسم ذلك على الإمام فما خرج فقوّسه فما كانت القوس فهو البعد من معدّل النها[ ر] فاحفظه فان كان ما بين الطولين اقلّ من تسعين فانظر عرض بلدك والبعد المحفوظ فانقص اقلّهما من اكثرهما وان كان اكثر من تسعين فزده على عرض البلد فما حصل من ايّ الوجهين كان فهو بعد البلد فخذ جيب تمامه واضربه في جيب تمام العمود واقسم المجتمع على ستّين وقوّس الخارج وانقص القوس من تسعين يكن الباقي قوس مسافة ما بين بلدك ومكّة فخذ جيبه واتّخذه امامًا ثمّ اضرب ستّين في جيب العمودواقسمه على الامام يخرج جيب قوس السمت فقوّس ذلك الخارج يكن قوس السمت وهو سمت مكّة ثمّ انظر فان كان بين طول مكّة وطول بلدك أقلّ من ص وعرض بلدك أكثر من بعد معدّل النهار فالسمت الّذي خرج هو من نقطة الجنوب الى الجهة الّتي منها شرقًا او غربًا وان كان عرض بلدك أقلّ من بعد معدّل النهار وكان بين الطولين أكثر من ص فالسمت الّذي خرج لك هو من نقطة الشمال الى جهة مكّة زادها الله تشريفًا وتكريمًا ٠

امّا اسباب معرفة القبلة قال القاضي : ارا[ ى ] أوضح[1] ما يتوصّل به الى معرفة القبلة ان تضع بلاطة مسطّحة محكمة السطح وضعًا موازيًا لافق التسطيح وتدير فيها دائرة بايّ قدر تريد وكلّما اتّسعت الدائرة كان أحسن فان اكتفيت برسم

1 In the manuscript اوصح



هذه الدائرة فخذها أحزال[2] أجزل وان استطعت ان تخطّ داخل هذه الدائرة دوائر كلّ دائرة لا يكون بينها و بين صاحبتها الاّ قدر الحاجز[3] الى دائرة صغيرة وهي الّتي تلي المركز يكون قطرها نصف سدس الكبرى فتمثّل ما بينهما من سطح البلاطة دوائر متوازية على مركز واحد في غاية الرقّة والإحكام لا يكون بين واحدة منهنّ والاخرى الاّ قدر ما يمكن من الحاجز[4] فتكون لك نقطة من البلاطة على محيط دائرة إمّا مرسومة وإمّا حاجزة ثمّ ضع في هذا المركز لهذه الدوائر قائمًا في غاية الإحكام في الطول يكون ارتفاعه نحو خمس قطر الدائرة العظمى او زائد بيسير ويكون طرفه الأعلى حادًا ، ثمّ ترصد الظلّ اذا وصل الى اوّل دائرة وهي العظمى في اوّل النهار وعلّمت في الدائرة العظمى علامة ثمّ لا يزال يقصر فتعلّم في الدائرة الأخرى الّتي تلي الأولى وكذلك اذا قصر في الأخرى وان امكنك رصد الارتفاع بربع صحيح او اسطرلاب فهو أحسن حتّى ينتهي الظلّ منتهاه في القصر ثمّ يبدأ الظلّ بالزيادة وتأخذ الارتفاع في النقصان وانت تعلّمه على الدوائر من الجهة الأخرى كما علّمت اوّلا وتأخذ تقابل الارتفاع بالارتفاع فان وافق عملك في النصف الاوّل من النهار للنصف الأخر فقد صحّ عملك ثمّ انظر الى الظلّ اذا وصل الى آخر دائرة وهي الكبرى فقد حصل لك على كلّ دائرة نقطتان فاقسم من كلّ نقطة الى صاحبتهما قسمين وعلّم علامة النصف فاذا انتظمت علامة التنصيف وعلامة التوقّف[5] و مركز الدوائر كلّها على خطّ مستقيم فذلك غاية التصحيح وان لم تنتظم فأعد العمل واخرج على علامات التنصيف خطًّا مستقيمًا الى المركز والى الدائرة الكبرى يكن ذلك خطّ نصف النهار واخرج عليه خطًّا قائمًا يكن ذلك خطّ المشرق والمغرب [ . . . ]

المثال : فإنّا أردنا معرفة قبلة تونس وكان الفضل بينهما في الطولين على ان مكّة عز وتونس ما مه والفضل له يه جيبه لد لح جيب تمام عرض مكّة نه[6] مو ضرب في جيبه ما بين الطولين وقسم الخارج على ستّين خرج لب يا كج قوّس ذلك خرج من ذلك لب كز وهي الفضلة الاولى وهي العمود والقوس الاولى احدب « ؟ »

2 In the manuscript وخذها

3 In the ms. الحاحز

4 In the manuscript الحاحز

5 Ms. add. وعلامة

6 Ms. قه

جيب تمامها وذلك ن لح وهو الاسفل ضربت جيب عرض مكّة وهو كب ط في ستّين خرج من ذلك ١٣٢٩[7] قسم على جيب تمام القوس الاولى وهو ن لح خرج كو يه قوّس ذلك خرج البعد من معدّل [ النهار] وهي القوس الثانية كه نز زيد مع القوس الثانية على تمام العرض الّذي هو ارتفاع رأس الحمل لأنّ الفضل بين الطولين أقلّ من تسعين درجة فكان الارتفاع نج ك خرج عط يز أخذنا جيبه فكان نح[8] نز يج وهو الجيب المحصّل ضربناه في جيب تمام القوس الثانية خرج بعد القسمة على ستّين مط مد ن قوّس ذلك خرج لج نو جيب تمامه وهو المحفوظ قسمنا جيب القوس الاولى على [ جيب ] المحفوظ [ وضربنا الخارج في ستّين ] خرج نز لط ن[9] قوّس ذلك خرج عج [ نز] لد نقص ذلك من تسعين بقي يو ب كو[10]

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7 Ms. و ح حح ا. The copyist has obviously mistaken the Arabic numerals and considered them to be written in abjad notation

8 Ms. مح

9 Ms. عج لد

10 Ms. يو كد



## APPENDIX 2: FROM CHAPTER 18 OF "SCRIPTUM ANTIQUUM SARACENI CUISDAM, DE DIUERSARUM GENTIUM ERIS, ANNIS AC MENSIBUS ET DE RELIQUIIS ASTRONOMIAE PRINCIPIIS" ACCORDING TO THE EDITION NÜREMBERG, 1549.[32]

*Ad sciendum rectitudinem orientum, et occidentum, et meridiei.*

Manifestius quo peruenitur ad sciendum meridiem et clarius est, ut ponas marmor planum decentis planicei, et situs aequidistantis orizonti plano, et reuolue in ipsa circulum cum quacunque quantitate uolueris, et quanto magis dilatatur eo erit melius. Si ergo contentus fueris descriptione huius circuli, tantum sufficiet tibi, et si poteris lineare intra hunc circulum, et super ipsius centrum circulum alium connexum ei, ita quod non sit inter *eos*[33] ambos, nisi quod minus tibi possibile est, de eo quod distinguit inter utrosque a contractu, ut sit distinguens quod est inter utrosque, quasi circulus tertius medius inter eos, et quo minus hoc feceris eo erit melius, et non cessabis lineare inter omnem circulum, circulum alium secundum quod dixi tibi, donec peruenias ad circulum paruum, cuius diameter sit medietas sextae maioris aut quasi illud. Assimulabitur ergo quod erit inter eos ambos, de superficie marmoris circulis aequidistantibus super centrum unum in extremitate paruitatis et decoris, ita ut non sit inter omnem circulum, et illum qui sequitur eum, nisi minus quod possibile est de distinguentibus, ergo erit omne punctum marmoris stans super circumferentia circuli aut signi, aut distinguentis. Deinde pones in centro horum circulorum, singulare erectum secundum angulos rectos cum magis uero situ et decore, et sit eius altitudo quasi quinta diametri circuli magni, aut parum augmentatior, et sit extremitas eius superior ad acumen quoddam tendens. Deinde considerabis umbram eius ante meridiem, ubi incipit umbra intrare in circulos, et non cures considerare eam cum est extra maiorem eorum. Cum ergo peruenerit cum coartatione ad latiorem circulorum, signabis super circumferentiam eius notam aduentus eius, deinde non cessabit umbra minorari. Quare permutatur de circulo, ad distinguens quod

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[32] We wish to express here our gratitude to Prof. J. Bastardas (University of Barcelona) for his revision of this text.

[33] *Eos*: ed. *nos*.

sequitur cum <est> extra ipsum. Signabis ergo illic notam deinde super illum quod sequitur ipsum. Et si possibile tibi fuerit cum hoc consideratio altitudinis cum quarta circuli uera, aut cum <astrolabio>[34], erit melius. Non ergo cessabit altitudo in quarta circuli addi, et umbra in marmore minui, donec peruenerit in paruitate ad finem sui status, et scies illud per essentiam super circumferentiam circuli super minorem, quo non fuit ante minor, et quasi stet illic. Signabis ergo stationis notam, deinde incipiet addi. Si ergo conuenerit ille status altitudini in additione, tunc iam uerificatum est, et si diuersificatur, tunc iam ingressus est in operationem error. Ergo subsiste in eo. Si ergo conuenerint acceptio umbrae in additione et altitudo in diminutione, et manifestum fuerit illud, tunc dimittas considerationem altitudinis, et eris solicitus de ratione umbrae. Quoties autem peruenerit cum extensione ad circulum, signabis illic notam, donec protendatur umbra extra a maiore circulorum. Dimitte ergo considerationem tunc, ergo iam prouenerunt tibi super omnem circulum duo puncta, et super circulum unum qui est minor circulorum nota una, et est nota stationis. Diuide ergo quod est inter omnes duas notas circuli in duo media, et signa notam medietatis. Cum ergo ordinantur notae mediationis, et nota stationis, et centrum circulorum, omnia super lineam unam rectam tunc illa est ultima <nota>[35] uerificationis. Et si diuersificantur, tunc non sunt cum termino ueritatis. Si ergo possibile est illud iterare, donec uerificetur secundum conditiones praedictas, fac, et si non sis contentus nota stationis, et centro circulorum, extrahe lineam rectam transeuntem per ea, et fac penetrare extremitatem eius usque ad circumferentiam circuli magni. Illa ergo erit linea meridiei accepta a septentrione ad meridiem, et extrahe a centro circulorum lineam rectam super hanc lineam orthogonaliter. Erit ergo haec linea ab oriente ad occidentem.

[...] Ad sciendum uero rectitudinem meridiei, scias longitudinem Metre, quae est sexaginta septem gradus ab occidente, aut sexaginta septem gradus, et medietas et latitudinem ipsius, quae est unius et viginti graduum, et medietas, aut duae tertiae. Et scias longitudinem regionis tuae, et ipsius latitudinem. Deinde multiplica sinum complementi latitudinis Metre in sinum eius, quod est inter duas longitudines, et diuide aggregatum per sexaginta, et quod egredietur arcuabis, et erit superfluitas longitudinalis, et est perpendicularis, serua ergo eam, et pone sinum complementi eius praelatum, et multiplica sinum latitudinis Metre in sexaginta, et diuide quod aggregatum est per praelatum, et quod egredietur, arcua illud, et arcus qui est, erit longitudo ab equatore diei. Ergo serua ipsam. Si ergo illud quod est inter duas longitudines fuerit minus nonaginta, tunc considera latitudinem regionis tuae, et longitudinem seruatam, et minue minorem earum de maiore ipsarum, et si fuerit plus nonaginta, tunc adde super latitudinem regionis, et quod prouenerit ex quolibet duorum modorum, est longitudo regionis. Accipe ergo sinum complementi eius, et multiplica in sinum complementi perpendicularis, et diuide aggregatum per sexaginta, et arcua quod egreditur, et minue arcum de nonaginta, et residuum erit arcus spacii, quod est inter regionem tuam, et Metram. Accipe ergo sinum eius et assume eum praelatum. Deinde multiplica sexaginta in sinum perpendicularis, et diuide illud per prelatum, et arcua quod egreditur, et erit arcus rectitudinis post rectitudinem Metre. Deinde considera si fuerit inter longitudinem Metre, et longitudinem regionis tuae minus nonaginta, et latitudo regionis tuae maior longitudine aequatoris diei, aut fuerit illud quod est inter duas longitudines plus nonaginta, tunc rectitudo quae egreditur tibi, est a puncto septentrionis ad partem Metre.

[34] *Astrolabio*: ed. *alio ab illa*. Our correction is based on the Arabic text.

[35] *Nota*: ed. *non*.